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THE IDENTIFICATION PROBLEM IN THE GAS-LIFT PROCESS MODELED BY THE SYSTEM OF DIFFERENTIAL EQUATIONS WITH DELAYED ARGUMENTS (DISCRETE CASE)*

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Abstract. In the paper the identification problem is considered on the basis of a mathematical model built for the gas-lift process. The method is proposed to determine the delays and parameters on the basis of a mathematical model established under the assumption that the pressure and flow in the bottom of the lifting pipe depends on the time delay and parameters of the pressure and flow.

Keywords: Gas-lift, delay argument, lifting pipe, differential equations, annual space.

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1 Introduction

At present, three main methods are used in the exploitation of oil fields: the first is the flowing method of oil production, the second is the gas-lift method, and the third is the method of exploiting oil wells with the help of downhole pumps. In the flowing method, oil comes to the surface as a result of high pressure in the reservoir. After the well has been flown for a certain period of time, the pressure in the oil reservoir decreases significantly, and oil can no longer rise to the surface under the influence of natural gas or layer water, which stops the oil flow process. In this case, the most effective method of operating an oil well is the gas-lift method. The substance of this method lies in the fact that when additional gas is pumped into the well, a gas-liquid mixture is formed at its bottom, which has lower specific gravity, which in turn allows oil to leave the well together with gas.

It should be noted that a large number of works by various authors Aliev & Ilyasov (1985); Guo (2011); Yudin et al. (2019); Mayhii (1974); Rashid et al., (2012) are devoted to the mathematical modeling of the gas-lift method. However, the creation of an adequate mathematical model remains an urgent problem. Note that different mathematical models have been proposed to describe adequately the gas-lift process Aliev & Mutallimov (2012); Barashkin & Samarin (2006). The peculiarity of the proposed model is that we are talking about the optimal control for the effective use of the gas-lift method. In Barashkin & Samarin (2006), such a problem was reduced to a linear-quadratic optimal control problem and an algorithm for its solution was

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developed. As a result, the optimal program trajectory and controls were found for this problem. After that, on the basis of the results of Charniy (1951); Magarramov & Gadjieva (2017) and Barashkin & Samarin (2006), the problem of constructing optimal stabilization around the program trajectory and control was solved, i.e. the problem of stabilization is solved. The model under consideration assumes that the pressure and flow rate at the bottom of the well are constant, which is reflected in the conditions at the bottom of the well. However, this is not always the case. The flow from the reservoir to the bottom of the well is a complex process that is very difficult to calculate and take into account in the model. Therefore, the flow and pressures at the lower end of the riser are functions that are derived from the annulus and formed through the formation, the relationship between which can be represented as difference equations. This approach was proposed in Rasulova (2022), which provides a large amount of gas-liquid mixture at the exit of the lift.

It should be noted that the volume flow and pressure of the pressurized gas in the lower part of the annulus can only affect the lower part of the tubing after a certain delay. Based on this assumption, the mathematical model describing the gas-lift process in Rasulova (2019) ultimately reduces to a differential equation with a retarded argument, the solution of which requires a special approach.

In contrast to Barashkin & Samarin (2006), a new mathematical model was proposed, assuming that the pressure and flow at the bottom of the riser pipe depend on the pressure and flow at the bottom of the annular area with an indefinite time delay and unknown parameters. On the basis of this model, an identification method was developed to determine the time delay and unknown parameters. For identification, statistical data of wells will be used as in Baigereyev et al. (2015).

2 Problem statement

In Aliev & Ilyasov (1985), the following system of linear partial differential equations is proposed as an approximate model of the gas-lift proses

$$\begin{cases} \frac{\partial P}{\partial t} = -\frac{c^2}{F}\frac{\partial Q}{\partial x} \\ \frac{\partial Q}{\partial t} = -F\frac{\partial P}{\partial x} - 2aQ \end{cases} \tag{1}$$

with initial

$$P(x,0) = P^0(x), \qquad Q(x,0) = Q^0(x)$$

boundary

$$P(0,t) = P_0(t), \qquad Q(0,t) = Q_0(t)$$

and additional matching conditions at $x = L$

$$\begin{array}{l} Q(L+0,t) = \alpha Q(L-0,t-\tau) \\ P(L+0,t) = \beta P(L-0,t-\tau). \end{array} \tag{2}$$

Here $P(x,t)$ and $Q(x,t)$ correspondingly are the pressure and mass flow of the gas injected into the annual space and gas-liquid mixture in the lifting pipe, $t \geq 0$, $x \in [0, 2L]$, τ certain parameter and

$$c = \begin{cases} c_1, & x \in (0,\, L) \\ c_2, & x \in (L,\, 2L) \end{cases}, F = \begin{cases} F_1, & x \in (0,\, L) \\ F_2, & x \in (L,\, 2L) \end{cases}, a = \begin{cases} a_1, & x \in (0,\, L) \\ a_2, & x \in (L,\, 2L) \end{cases}$$

This system can be easily reduced to the system of ordinary differential equations by dividing L into n equal parts $l = L/n$. In this case

$$x_i = \frac{L}{n} i, \qquad i = \overline{0, 2n}$$

$$P(x_i, t) = P_i(t), \quad Q(x_i, t) = Q_i(t)$$

$$\left.\frac{\partial P}{\partial x}\right|_{x = x_i} = \overset{\bullet}{P}_i(t) \approx \frac{P(x_i, t) + P(x_{i-1}, t)}{l} = \frac{1}{l}\left[P_i(t) - P_{i-1}(t)\right],$$

$$\left.\frac{\partial Q}{\partial x}\right|_{x = x_i} = \overset{\bullet}{Q}_i(t) \approx \frac{Q(x_i, t) + Q(x_{i-1} + t)}{l} = \frac{1}{l}\left[Q_i(t) - Q_{i-1}(t)\right],$$

If take into account its expressions, initial and matching conditions and their relation for the lifting pipe

$$\bar{Q_n}(t) = \alpha Q_n(t - \tau),$$

$$\bar{P_n}(t) = \beta P_n(t - \tau), \;\; i = \overline{1, n} \tag{3}$$

then we get a system of ordinary differential equations for the annual space from (1). That is, for $i = \overline{1, n}$

$$\begin{cases} \dot{P}_1(t) = -\frac{c_1^2}{F_1 l} Q_1(t) + \frac{c_1^2}{F_1 l} Q_0(t), \\ \dot{Q}_1(t) = -\frac{F_1}{l_1} P_1(t) + \frac{F_1}{l_1} P_0(t) - 2a_1 Q_1(t), \\ \dot{P}_2(t) = -\frac{c_1^2}{F_1 l} Q_2(t) + \frac{c_1^2}{F_1 l} Q_1(t), \\ \dot{Q}_2(t) = -\frac{F_1}{l_1} P_2(t) + \frac{F_1}{l_1} P_1(t) - 2a_1 Q_2(t), \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \dot{P}_n(t) = -\frac{c_1^2}{F_1 l} Q_n(t) + \frac{c_1^2}{F_1 l} Q_{n-1}(t), \\ \dot{Q}_n(t) = -\frac{F_1}{l} P_n(t) + \frac{F_1}{l} P_{n-1}(t) - 2a_1 Q_n(t). \end{cases} \tag{4}$$

Here the functions $Q_0(t)$, $P_0(t)$ are the volume of the injected gas to the annular domain and its use correspondingly, and gas-lift process is controlled by these functions. It is known that $Q_n(t)$ and $P_n(t)$ are the values of these functions correspondingly, in the bottom of the well. It is clear that the pressure and use formed that in the bottom of the lifting pipe of the well depends on their values in the annular at the argument t with the delay τ. Considering (3) for the lifting pipe we get the following equations

$$\begin{cases} \dot{P}_{n+1}(t) = -\frac{c_2^2}{F_2 l} Q_{n+1}(t) + \frac{c_2^2}{F_2 l} \alpha Q_n(t - \tau), \\ \dot{Q}_{n+1}(t) = -\frac{F_2}{l} P_{n+1}(t) + \frac{F_2}{l} \beta P_n(t - \tau) - 2a_2 Q_{n+1}(t), \\ \dot{P}_{n+2}(t) = -\frac{c_2^2}{F_2 l} Q_{n+2}(t) + \frac{c_2^2}{F_2 l} Q_{n+1}(t - \tau), \\ \dot{Q}_{n+2}(t) = -\frac{F_2}{l} P_{n+2}(t) + \frac{F_2}{l} P_{n+1}(t - \tau) - 2a_2 Q_{n+2}(t), \\ - \\ \dot{P}_{2n}(t) = -\frac{c_2^2}{F_2 l} Q_{2n}(t) + \frac{c_2^2}{F_2 l} Q_{2n-1}(t), \\ \dot{Q}_{2n}(t) = -\frac{F_2}{l} P_{2n}(t) + \frac{F_2}{l} P_{2n-1}(t) - 2a_2 Q_{2n}(t). \end{cases} \tag{5}$$

Thus we arrive at the system of the first order differential equations (4), (5). The corresponding initial conditions have the form:

$$P_i(0) = P_i^0, \quad Q_i(0) = Q_i^0 \qquad i = \overline{1, 2n}. \tag{6}$$

Then from expressions (4) and (6) we have

$$x(t) = [P_1(t), Q_1(t), P_2(t), Q_2(t), \ldots, P_n(t), Q_n(t), P_{n+1}(t), Q_{n+1}(t), ..., P_{2n}(t), Q_{2n}(t)]',$$
$$x^0 = \left[P_1^0, Q_1^0, P_2^0, Q_2^0, \ldots, P_n^0, Q_n^0, P_{n+1}^0, Q_{n+1}^0, ..., P_{2n}^0, Q_{2n}^0, \right]',$$
$$u(t) = [Q_0(t), P_0(t)]'$$

following Cauchy problem.

$$\dot{x}(t) = Ax(t) + Bx(t-\tau) + Gu(t), \tag{7}$$

$$x(t) = x^0(t),\ t \in [-\tau, 0]. \tag{8}$$

Here the matrices A, B, G are defined as in [11].

Now let's proceed to the decomposition of problem (7) and (8).

$$x(t) = [x_1'(t), x_2'(t), ..., x_n'(t), x_{n+1}'(t), \ldots, x_{2n}'(t)]'$$

$$x_i(t) = \begin{bmatrix} P_i(t) \\ Q_i(t) \end{bmatrix}, \qquad i = \overline{i,\, 2n}, \qquad x_i^0 = \begin{bmatrix} P_i^0 \\ Q_i^0 \end{bmatrix},$$

$$A_1 = \begin{bmatrix} 0 & -\frac{c_1^2}{F_1 l} \\ -\frac{F_1}{l} & -2a_1 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 0 & -\frac{c_2^2}{F_2 l} \\ -\frac{F_2}{l} & -2a_2 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 & \frac{c_1^2}{F_1 l} \\ \frac{F_1}{l} & 0 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} 0 & \frac{c_2^2}{F_2 l} \\ \frac{F_2}{l} & 0 \end{bmatrix},$$

$$u(t) = \begin{bmatrix} P_0(t) \\ Q_0(t) \end{bmatrix}, \qquad V(\alpha, \beta) = \begin{bmatrix} 0 & \frac{c_2^2}{F_2 l}\alpha \\ \frac{F_2}{l}\beta & 0 \end{bmatrix},$$

$$\begin{cases} x_1'(t) = A_1 x_1(t) + B_1 u(t) \\ x_2'(t) = A_1 x_2(t) + B_1 x_1(t) \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ x_n'(t) = A_1 x_n(t) + B_1 x_{n-1}(t) \\ x_{n+1}'(t) = A_2 x_{n+1}(t) + V(\alpha, \beta)\, x_n(t-\tau) \\ x_{n+2}'(t) = A_2 x_{n+2}(t) + B_2 x_{n+1}(t) \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ x_{2n}'(t) = A_2 x_{2n}(t) + B_2 x_{2n-1}(t) \end{cases} \tag{9}$$

$$x_i(0) = x_i^0,\ i = \overline{1, n},\ i = \overline{n+2,\, 2n} \tag{10}$$

$$x_{n+1}(t) = x_{n+1}^0(t),\ t \in [-\tau, 0].$$

Solution of problem (9)-(10).

The solution of problem (9)- (10) in continuous case and the investigation of the corresponding identification problem is given in [12]. Here we consider this problem (9) - (10) in discrete case. First of all

$$\dot{x}_i(t) = A_1 x_i(t) + B_1 u(t),\ x_i(0) = x_i^0 i = 1,\, k \tag{11}$$

Let's discretize the equations.

For this, we make the following approximation:

$$\dot{x}_i(t) \approx \frac{x_i(t_{k+1}) - x_i(t_k)}{h}.$$

Dividing the interval $[0,\, T]$ into M equal parts

$$\begin{aligned} &t_0 = 0 \\ &t_k = t_0 + h \cdot k \\ &h = \tfrac{T}{M}. \end{aligned}$$

Here, M is a natural number. Considering this in (11) we obtain

$$x_i(t_{k+1}) = x_i(t_k) + h\ [A_1\, x_i(t_k) + B_1 u\,(t_k).] \tag{12}$$

Assume that $u(t) = u_0 = \begin{bmatrix} P_0 \\ Q_0 \end{bmatrix}$ are constant vectors. Then from (12) we get

$$x_1(t_1) = x_1(t_0) + h\,[A_1 x_1(t_0) + B_1 u\,(t_0)] = [1 + hA_1]\, x_1(t_0) + hB_1 u(t_0)$$

$$\begin{aligned} &x_1(t_2) = [1 + hA_1]\, x_1(t_1) + hB_1 u(t_1) = [1 + hA_1]^2\, x_1(t_0) + [1 + hA_1]\ hB_1 u(t_0) + hB_1 u(t_1) = \\ &= [1 + hA_1]^2\, x_1(t_0) + hB_1\,[(1 + hA_1)u(t_0) + u(t_1)] \end{aligned}$$

$$x_1(t_3) = [1 + hA_1]x_1(t_2) + hB_1 u(t_2) = [1 + hA_1] \cdot [1 + hA_1]^2 x_1(t_0) + hB_1[(1 + hA_1)u(t_0) + u(t_1)]$$

$$+hBu(t_2) = [1 + hA_1]^3 x_1(t_0) + hB_1[1 + hA_1^2 u(t_0) + (1 + hA_1)u(t_1) + u(t_2)] =$$

$$= [1 + hA_1]^3 x_1(t_0) + hB_1[(1 + hA_1^2 u(t_0) + (1 + hA_1 u(t_1) + u(t_2)]$$

$$x_1(t_4) = [1 + hA_1]^4 x_1(t_0) + hB_1[(1 + hA_1)^3 u(t_0) + (1 + hA_1)^2 u(t_1) + (1 + hA_1)u(t_2) + u(t_3)]$$

....

$$x_1(t_k) = [1 + hA_1]^k\, x_1(t_0) + hB_1(1 + hA_1)^{k-1} u(t_0) + (1 + hA_1)^{k-2} u(t_1) + \ldots +$$

$$(1 + hA_1)u(t_{k-2}) + u(t_{k-1}) = [1 + hA_1]^k\, x_1(t_0) + hB_1 \sum_{i=0}^{k-1} (1 + hA_1)^{k-1-i} u(t_i).$$

By the same way

$$x_2(t_k) = [1 + hA_1]^k\ x_2(t_0) + hB_1 \sum_{i=0}^{k-1} (1 + hA_1)^{k-1-i} x_1(t_i)$$

$$x_3(t_k) = [1 + hA_1]^k\ x_3(t_0) + hB_1 \sum_{i=0}^{k-1} (1 + hA_1)^{k-1-i} x_2(t_i)$$

....

$$x_n(t_k) = [1 + hA_1]^k \; x_n(t_0) + hB_1 \sum_{i=0}^{k-1} (1 + hA_1)^{k-1-i} x_{n-1}(t_i).$$

Now for equation in (9) after the approximation we obtain

$$x'_{n+1}(t) = A_1 x_n(t) + B_1 x_{n-1}(t - \tau) \tag{13}$$

$x_{n+1}(t_k - \tau)$. We use the Taylor series to calculate the equation (13)

$$x_{n-1}(t_k - \tau) = x_{n-1}(t_k) - x'_{n-1}(t_k) \cdot \tau + \ldots \tag{14}$$

Here

$$x'_{n-1}(t_k) \approx \frac{x_n(t_{k+1}) - x_{n-1}(t_k)}{h}$$

If consider the relation

$$x_{n+1}(t_{k+1}) = x_{n+1}(t_k) + h \left[A_1 x_n(t_k) + B_1 \left[x_{n-1}(t_k) - \tau \frac{x_{n-1}}{(} t_{k+1}) - x_{n-1}(t_k) h \right] \right]$$

$$= x_{n+1}(t_k) + hA_1 x_n(t_k) + hB_1 x_{n-1}(t_k) - B_1 \tau (x_{n-1}(t_{k+1}) - x_{n-1}(t_k)). \tag{15}$$

then

$$\dot{\underset{i}{x}}(t) = A_2 x_i(t) + B_2 x_{i-1}(t), \qquad i = \overline{n+2,\, 2n}, \tag{16}$$

Now we consider (9). Let's solve equation (16) according to (11). Then

$$\begin{cases} x_{n+2}(t_k) = [1 + hA_2]^k \; x_{n+2}(t_0) + hB_2 \sum\limits_{i=0}^{k-1} (1 + hA_2)^{k-1-i} \, x_{n+1}(t_i), \\ \ldots\ldots\ldots\ldots \\ x_{2n}(t_k) = [1 + hA_2]^k \; x_{2n}(t_0) + hB_2 \sum\limits_{i=0}^{k-1} (1 + hA_2)^{k-1-i} \, x_{2n-1}(t_i). \end{cases}$$

Obviously,

$$x_{2n}(t_k) = F_{2n}(\alpha, \beta, \tau, t_k),$$

Thus the function $x_{2n}(t_k)$ depends on parameters α, β, τ. This means that for each value of parameters α, β and τ we can find a certain solution $x_{2n}(t_k)$, which corresponds to the flow rate at the surface of the well.

3 Identification of unknown parameters

Let's assume that I = [0; 1]is a vector. Then it is clear that the well flow rate can be found in the form

$$Q_{2n}(\alpha, \beta, \tau, T) = \mathrm{I} \cdot x_{2n}(T). \tag{17}$$

This means that for each parameter α, β and $\tau$the flow rate can be determined, found by formula (17).

Now consider the problem of finding parameters α, β and τ , i.e. identification.

Let us assume that the statistical values of the injected gas $Q_0^{S,st}$ and the corresponding flow rate $Q_{2n}^{s,st}$ are known from the history of the studied gas-lift well. Then it is found based on statistical data.

$$u^{S,st}(t)=\begin{bmatrix} P_0^{s,st} \\ Q_0^{s,st} \end{bmatrix}.$$

Having determined the function, it is possible to solve the system of differential equations (9), (10) for the specified parameters α, β and τ also find the debit $Q_{2n}^{s}\left(\alpha,\beta,\tau,T\right)$ using formula (17). Here $s=\overline{1,\ k}$, k is the total number of statistics. Build functionality using these solutions and $Q_{2n}^{s,st}$ statistical data.

$$\mathrm{I}(\alpha,\beta,\tau)=\sum_{s=1}^{k}\left[Q_{2n}^{s}(\alpha,\beta,\tau,T)-Q_{2n}^{s,st}\right]^2. \tag{18}$$

The problem is to find such values of the parameters α, β and τ so that the function (18) takes the minimum value.

This means that the values of parameters α, β and τ must be found in such a way that the solution of the system of differential equations (9), (10) gives the minimum value of function (18).

Thus, we obtain the optimization problem (9), (10), (18). To solve this problem, you need to find the gradient of the function $\mathrm{I}(\alpha,\beta,\tau)$ and solve the system of equations so that it becomes equal to 0, and find the parameters α, β and τ. From the functional (18), it can be seen that it is practically impossible to analytically find the gradient vector of the functional $\mathrm{I}(\alpha,\beta,\tau)$, therefore, as in the article Sakharov & Mishchenko (1985), the expression from Karchevsky & Dedok (2018) can be used to find the derivatives $\frac{\partial \mathrm{I}}{\partial\alpha}$, $\frac{\partial \mathrm{I}}{\partial\beta}$ and $\frac{\partial \mathrm{I}}{\partial\tau}$:

$$\begin{array}{l}
\frac{\partial \mathrm{I}(\alpha,\beta,\tau)}{\partial\alpha}\approx\begin{cases} 0,\ if\begin{cases} I\left(\alpha,\beta,\tau\right)\le I(\alpha+h_\alpha,\beta,\tau) \\ I\left(\alpha,\beta,\tau\right)\le I(\alpha-h_\alpha,\beta,\tau)\end{cases} \\ \frac{\mathrm{I}(\alpha+h_\alpha,\beta,\tau)-\mathrm{I}(\alpha-h_\alpha,\beta,\tau)}{2h_\alpha}\end{cases} \\
\frac{\partial \mathrm{I}(\alpha,\beta,\tau)}{\partial\beta}\approx\begin{cases} 0,\ if\begin{cases} I\left(\alpha,\beta,\tau\right)\le I(\alpha,\beta+h_\beta,\tau) \\ I\left(\alpha,\beta,\tau\right)\le I(\alpha,\beta-h_\beta,\tau)\end{cases} \\ \frac{\mathrm{I}(\alpha,\beta+h_\beta,\tau)-\mathrm{I}(\alpha,\beta-h_\beta,\tau)}{2h_\beta}\end{cases} \\
\frac{\partial \mathrm{I}(\alpha,\beta,\tau)}{\partial\tau}\approx\begin{cases} 0,\ if\begin{cases} I\left(\alpha,\beta,\tau\right)\le I(\alpha,\beta,\tau+h_\tau) \\ I\left(\alpha,\beta,\tau\right)\le I(\alpha,\beta,\tau-h_\tau)\end{cases} \\ \frac{\mathrm{I}(\alpha,\beta,\tau+h_\tau)-\mathrm{I}(\alpha,\beta,\tau-h_\tau)}{2h_\tau}\end{cases}
\end{array} \tag{19}$$

where h_α,h_β,h_τ is quite a small step.

$$\begin{array}{l}
\frac{\partial \mathrm{I}(\alpha,\beta,\tau)}{\partial\alpha}=0, \\
\frac{\partial \mathrm{I}(\alpha,\beta,\tau)}{\partial\beta}=0, \\
\frac{\partial \mathrm{I}(\alpha,\beta,\tau)}{\partial\tau}=0.
\end{array} \tag{20}$$

Then we can find the unknown parameters α, β and τ by solving the system of equations. Thus, the following algorithm can be proposed to find the parameters α, β and τ.

4 Algorithm

Step 1. Input the constants C, F, a, L and functions $Q_0(t)$and $P_0(t)$ specified in (1) - (3).
Step 2. Input the constants Q_0^i and $Q_{2n}^i\,(i=\overline{1,\ k})$.
Step 3. Solve system of equations (9), (10) and find the values $Q_{2n}^i(\alpha,\beta,\tau,Q_0^i,T)$.
Step 4. Form (18).
Step 5. Solve system of equations (20) using relations (19), and find the parameters α, β and τ.

Step 6. If the conditions

$$\left|\frac{\partial \mathrm{I}(\alpha,\beta,\tau)}{\partial \alpha}\right| < \varepsilon, \ \left|\frac{\partial \mathrm{I}(\alpha,\beta,\tau)}{\partial \beta}\right| < \varepsilon, \ \left|\frac{\partial \mathrm{I}(\alpha,\beta,\tau)}{\partial \tau}\right| < \varepsilon.$$

for the sufficiently small number $\varepsilon > 0$ is satisfied then stop the calculation, otherwise take to $h_\alpha, h_\beta, h_\tau$ and go the step 3.

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